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ELECTRIC RADIATION FROM WIRES.

Electric Waves. Being an Adams Prize Essay in the University of Cambridge. By H. M. Macdonald, M.A., F.R.S., Fellow of Clare College. Pp. xiii + 200. (Cambridge: University Press, 1902.) Price 10*s*.

THE essay under review consists essentially of two parts. In one of them the author aims at a re-statement of electrodynamic theory in a manner which will avoid what he considers to be the difficulties of the existing dynamical expositions. The other part contains new developments relating to the mode of propagation of electric radiation, its emission and absorption by resonating wire circuits, and the dynamical laws of its diffraction by obstacles.

In illustration of the power of the mathematical analysis that is developed in the latter part, it may be mentioned that the general dynamical problem of diffraction at the edge of a perfectly conducting (*i.e.* totally reflecting) prism is solved in a few pages at the end of the book (Appendix D) by a method which admits of extension to any transparent or metallic prism the optical constants of which are known. The only case of diffraction in which a rigorous dynamical solution had been previously obtained is that of the straight edge of a perfectly conducting plate, which is the special case of a prism of vanishing angle; this had been reached through intricate analysis by Poincaré and by Sommerfeld, and the result is now often reproduced as a new departure in mathematical physics applied to problems in theoretical optics. The very elegant treatment in terms of Bessel functions that is brought to bear by Mr. Macdonald will remind readers of a previous successful application of essentially the same analysis, namely to the verification of Mr. W. D. Niven's beautiful functional solution of the problem of electric distribution on the general type of conductor bounded by two intersecting spheres, which was published some years ago in the *Proceedings* of the Mathematical Society. Features of much interest are bound to arise in the theoretical character of the diffraction at the edge of a transparent or metallic prism of known index; it is to be hoped that the author will not be deterred by some inevitable complexity of computation from following out in detail this natural extension of his results.

When disruptive electric disturbances take place in a material system, their energy is, in the ordinary course, dissipated by electric radiation into space, in so far as it is not degraded into heat by resistance. That any other state of affairs could exist has not been hitherto contemplated, though it has been known by experience that an electric vibrating system like the ring resonator of Hertz could go on oscillating for very many thousands of periods without much loss. The author claims that it is possible theoretically to have electric vibrating systems absolutely permanent, which would last for ever so far as radiation is concerned; that if electric waves are introduced into a nearly complete wire circuit, and if the ends are then connected so as to make the circuit a complete ring, a portion of the wave-motion will settle down into a steady state in the circuit and run round and round for ever, assuming, of course, that the circuit is perfectly conducting; that as such waves can only enter through the ends, so the only way of dissipating them is by cutting the circuit and allowing them to escape from the ends. This, even if it is not valid for thick anchor rings, is certainly practically correct for thin wires; and such systems in which electric oscillations are going on thus radiate mainly from the ends or points of the wires. The nature of the beam of radiation which issues from the end of a straight wire is here investigated theoretically, the form obtained for the wave-fronts around the end being shown to be in close accord with the observations of Birkeland and Sarasin. Fortified with this theoretical analysis, we can form a more vivid and confident idea of how exposed metallic points like those of lightning conductors may gather up stray radiations in the surrounding space, which may then be passed down around a system of properly attuned loops forming nearly closed circuits in the lower part of the wire, in each of which a selected period can be intensified by resonance and tapped off through a relay system into an appropriate recorder; and we can even imagine that the direction from which an incident train of disturbances comes may be estimated from the orientation of the plane of the resonating loop which responds to it most intensely.

The whole theoretical discussion is founded on, and in turn elucidates, an extension of the ancient electric dogma of the power of points into the new field of electric radiation. Closed electric circuits can be placed in relation of radiation and absorption with the surrounding æther, after the manner of radiating atoms in temperature equilibrium, by narrow breaks or attached spikes The subject is far from being exhausted; for example, the more complex and probably far more difficult problem suggests itself to compare the radiation that must escape from a sharp bend in the wire carrying the waves with the radiation issuing from its open end. From the standpoint of present interests, the theoretical elucidation of the circumstances on which depend the free periods of resonators of the Hertz pattern formed of simple wire rings with or without knobs at the ends, to which close attention is also devoted in the book, is hardly as important as this other related question of the theoretical conditions governing the emission and absorption of radiation from wire circuits and networks.

The periods of free electric surging in the dielectric sheets of various forms of condensers are also discussed; the correction for the open edge of a flat condenser is determined, expressed in the form that by adding a slip of certain breadth to the plates all round the edge the electric field between them may be taken as uniform right up to it. The result comes, of course, from application of the general principles of the mode of analysis applied in acoustics by Helmholtz in 1859 to the correction for the open ends of organ pipes.

As developed by our author, the key to the discussion of the oscillations and their free periods, in open wire circuits, lies in the determination of the radiation from the end part of a straight wire when standing electric

waves are surging along its surface. This provides a knowledge of the ratio in which the distance of the open end from the nearest node falls short of half a wave; and, the other successive nodes being practically equidistant, it thus affords a knowledge of the free periods in terms of the length of the wire. Finite curvature of the wire does not sensibly affect things; this was elucidated very clearly by Pocklington in 1897,[1] and his analytical device for replacing the electrification and electric flow on the wire by a series of changing electric doublets situated along it, the fields of disturbance of which are simply expressible, is here largely employed. Consider, in fact, a system of doublets of moment σ (in the magnetic sense) per unit length distributed along the length s of the wire; they are equivalent to a current of intensity $d\sigma/dt$, a charge of line-density $-d\sigma/dx$, and two point-charges $-\sigma_1$ and $+\sigma_2$ at the ends; as the true current vanishes at the ends, σ must be constant there, and so will vanish too. This kind of theory leads very directly, in § 67, to the character of the forced oscillation on the wire that is established by an impressed magnetic field in the surrounding region, which is symmetrical and therefore ranged in circles around the wire as axis. Each infinitesimal ring of impressed alternating magnetic force is propagated out directly into wider rings until it meets the point of the wire under consideration, but *in addition* it travels to the open end of the wire and thence down its length, the signs being such that the two parts cancel at the end; the amplitudes of these two interfering systems of rays of magnetic force are not attenuated with increase of the distance traversed, because each point of the wire is equidistant from all elements of the source. It is their interference that constitutes the standing waves on the wire. We have here rings of magnetic disturbance radiated from the outside sources, converging on the wire through its open end, and travelling down it; it would appear that the author's restriction to symmetry may largely be dispensed with. The conditions are now reversed, and a system of standing oscillations on the wire pouring out radiation into space is contemplated; that occurs only through open ends, the oscillatory surging on the perfectly conducting wire elsewhere being capable of adjusting itself locally, like waves on a musical cord, without having to constrain any radiation. If we know the distribution of the radiation from the open end of a straight wire, over the infinite sphere, we can, by reversal of the motions and treating the infinite spherical surface as a region of sources of disturbance, deduce by the previous analysis the positions of the nodes on the wire. In applying this method, the author considers (§ 78), for reasons not obvious, that an open end radiates uniformly over the hemisphere in front of it.[2] In the discussion of the Hertzian wire resonator which follows, the two contiguous ends are taken to constitute a Hertzian oscillating doublet, and this determines the required distribution of radiation at infinite distance; the reversed radiation is supposed to affect the two ends independently. One feels more confidence here than in the previous case of a single end; and the results are, in fact, in very close agreement with experimental measurements by Sarasin and de la Rive. The modification arising from arming the ends with small balls or plates is also gone into.

The author's verification of the known form of the wave-fronts near the open end of a wire, namely confocal paraboloids with focus at that end, also comes from the reversed motion as above. It appears that this result holds whatever be the distribution of the radiation over the infinite sphere, the magnetic force around the end being of the form $A \tan\frac{1}{2}\theta$. The author adverts to the transverse wave-fronts travelling along the wire towards the end and finally bending round near the end into paraboloids as it is approached; the wave-front may be considered as detained on the wire because the magnetic force is cyclic around the wire and could not be cyclic if the front escaped into free space. In fact, the value of the magnetic force above given obviously satisfies this necessary condition, its circulation $2\pi r \sin\theta . A \tan\frac{1}{2}\theta$ being equal to $4\pi A r$ along the wire and equal to zero along its prolongation; the current in the wire near the end is thus Ar. We have, therefore, only to show that the characteristic equation of a magnetic field disposed in circles around the wire is satisfied; and this is so, for by the Amperean relation it leads to a longitudinal component Z of electric force proportional to r^{-1}, which is of the right form, being near the end practically $e^{\iota\kappa r}/r$, which satisfies the equation $\nabla^2 Z + \kappa^2 Z = 0$. The transverse component of the electric force is similarly found to be proportional to $-r^{-1}\tan\frac{1}{2}\theta$; thus the resultant force is in the direction bisecting the angle between r and the direction of the wire produced, and is therefore tangential to parabolic wave-fronts as above stated, being wholly transverse close to the wire. There is some temptation to imagine the wings of the parabolic part as advancing towards each other and forming a narrow neck which is finally nipped through, the main part of the front then going off as a plane sheet of radiation, while the other part retreats back into the wire and gives rise to a reflected wave, somewhat in the manner described by Hertz ("Electric Waves," p. 144) for the case of an oscillating doublet.[1] For free oscillations on a wire with two ends, the radiation is, however, sideways.

The circumstance that the general features of some of the author's conclusions can be traced by simple reasoning, as he himself indicates, does not, of course, detract from their value or novelty; it rather tends to confirm the validity of the powerful mathematical analysis to the results of which they are a first approximation, and should stimulate similar inquiry as regards the other part of his results. That this type of analysis is yet destined to point the way into the heart of other important problems in mathematical physics there can be no doubt; now that spherical and ellipsoidal forms have received such

[1] *Proc.* Camb. Phil. Soc. In this powerful paper, the radiation from a complete circular wire comes in evidence, in a second-order approximation, through a very slight damping of the free oscillations. On the view above described, there should be no such effect; yet, on the other hand, the electricities can be separated to the two sides of the ring by an electric field, and should surge back in vibratory manner when released.

[2] It appears that the assumption of a considerably different law would not much affect the result.

[1] Mr. Macdonald informs me that this view is supported by the graph of his second approximation in § 77.

abundant attention, it is much to have a method that can deal in comparative simplicity with edges and prisms and cones.

The evidence is closing in more and more rigorously that the medium which transmits electrical and radiant effects must either completely accompany matter in bulk in its movements or else be entirely independent of such movements. If we adopt the latter hypothesis, to which theoretical considerations strongly point, and we still consider the æther to be something possessing translatory inertia, the nature of its kinetic energy will be entirely at our disposal as regards interpretation.

The author's order of exposition, in the theoretical chapters of this book, first develops the equations for the free æther, in terms of a vector potential; these are naturally purely vibrational; then the disturbance of electricity, which is really the exciting source of the phenomena, is introduced by adding the electric flux $-4\pi(u, v, w)$ to the expression $c^{-2}\frac{d^2}{dt^2}(F, G, H)$, which by these equations of propagation is equated to $\nabla^2(F, G, H)$. In other words, the elements of current are each of them introduced as a simple intrinsic pole of the vector potential, which in other respects obeys the purely vibrational equations for the æther of empty space. These equations, as solved by the Poisson analysis suitable for such cases, represent disturbances travelling out from the poles in the known manner of simple compact propagation, at any rate in all cases where the phenomena are periodic. The electric flux thus introduced is here named the convection current, presumably because it is afterwards going to be considered as arising solely from the motion of electric charges or ions; in the analysis of Appendix C it is the motion of a volume density. The significant remark now follows that

"the assumption is implicitly involved that Maxwell's æthereal displacement current is independent of the motion of the æther, if there is such a motion."

Does this mean that it belongs to the æther, but yet is disconnected from it so that it is left behind if the æther moves on? One is tempted to amend the last phrase and make it read, "therefore there is no such motion."

However this may be, practically it comes to the same thing; in the next chapter, the æthereal part of the total current is taken not to depend on the motion of the æther, but the convection current does depend on the motion of the matter. This leads, as is known, to Fresnel's formula for velocity of optical propagation in moving material media, and to the law of astronomical aberration of light; and the author's *dictum*, above quoted, has already *postulated* that it is not to affect the phenomena whether the æther moves or not.

The reluctance shown by the author to considering the æther as stationary in space is based mainly, it appears, on the ground that a field of magnetic force must be concerned with motion in the æther, so that if that medium were otherwise at rest, waves of radiation would be convected by a magnetic field. This is known not to be the case to any recognisable extent; and it is here explained that the magnetic motions are only a part of the disturbance, there being other latent motions in the æther which may exactly compensate. But, on the other hand, the objection is not essential; for magnetic energy may not be energy of simple translation, while if it is so, the velocity need not be of detectable magnitude provided the inertia is sufficiently great. And in the latter case these other latent motions would surely be themselves magnetic. This consideration points to retaining the most precise and directly presentable scheme, until it is definitely proved to be too narrow.

In the body of the book, the mathematical analysis is developed from the foundation of the circuital laws of Ampère and Faraday, as translated into simple analytical form, and rendered self-consistent by the introduction of displacement currents, originally by Maxwell. In Appendix C, these relations are fitted into a purely dynamical frame. They are derived from potential and kinetic energy functions

$$T = \tfrac{1}{2}\iiint\left(F\frac{df}{dt} + G\frac{dg}{dt} + H\frac{dh}{dt}\right)dx\,dy\,dz,$$

$$W = \tfrac{1}{2}\iiint(Xf + Yg + Zh)dx\,dy\,dz;$$

but the other Maxwellian expression, more like ordinary kinetic energy,

$$T = \frac{1}{8\pi}\iiint(\alpha^2 + \beta^2 + \gamma^2)dx\,dy\,dz,$$

is considered to be unwarranted. This must mean that the kinetic energy is distributed in the medium according to the first form of integral, and that the second, though it gives the right total amount throughout all space, does not express its correct distribution in space. This is a question as to matter of fact. Not to press the point that the element of energy given by it is not essentially positive, the first specification might be thought to imply that (F, G, H) can be expressed in terms of the local conditions alone; but the only formula for this vector that is given is a volume integral depending on the state of the whole electric field. One result of the change is, of course, that the Poynting vector for the flux of energy must be modified, so that near the vibrator the paths of rays would be altered; when the waves become plane it does not matter.

If we turn to the mathematically analogous (but physically different) hydrodynamic theory by way of illustration, the kinetic energy of a fluid containing vortex lines can be expressed in terms of the vorticity by a cognate integral involving the vortex distribution alone, and the behaviour of the vortexes might be deduced from it, abstraction being made altogether of the fluid in which they exist. So the phenomena of the electric currents would be developed with abstraction altogether of the æther in which they subsist; except that, unfortunately, when the field is not steady, all the æther has to be filled with fictitious æthereal current which is not electric flux at all, *or else* all effects of true electric flux have to be considered as propagated in time. This is, in fact, the course of the actual historical development of the theoretical electrodynamics of ordinary steady

electric currents in the hands of Ampere and his successors; no mention need be made of the æther until electric radiation begins to play a sensible part, either in the establishment of the field or in the draining off of its energy, or until motion of the electric charges is contemplated. In the latter case, it would appear that we have either to take the æther to be at rest or to say with our author that it behaves as if it were so.

The analogy has here been drawn (which Mr. Macdonald doubtless would not allow) between the analysis of the interactions of electric currents in an æther which is intangible and that of vortical smoke-rings in an atmosphere which is invisible. In each case, one would try to avoid assuming unnecessary properties of the medium. And it is only fair to admit that the properties of electric currents have actually been discovered in this way, while without discussing the fluid we should hardly have made much progress with the more fugitive vortexes.

The process of arriving at wider and wider points of view by successive stages of generalisation from an initial hypothesis is a familiar and fruitful one in theoretical physics; though in these latter times the logical and philosophical merits of the converse process of discarding from our knowledge all colorable images or analogies, in favour of bare mathematical expression of the relations of the unknown quantities which are symbols for entities on which we do not wish to dogmatise at all—of which we, in fact, know intrinsically no more than we do about the most common objects around us—has also been amply enforced. Yet in successful instances of this latter procedure, the retort seems open that the hypothesis or analogy has not been dispensed with until it has effectively disclosed of what type the said relations were to be. It very likely arises from want of familiarity with Mr. Macdonald's point of view that a doubt suggests itself as to whether we have not here a case, if not of kicking away the ladder before the passenger has arrived at the top, at any rate of removing the supporting framework before the ties and struts of the permanent structure have become entirely consolidated.

Much in these remarks has assumed a critical form, because after pointing out the excellences that can be enjoyed by consulting the work itself, it would appear that a reviewer could do best service by discussing the matters that are not so clear. Other more detailed topics might be specified which require further consideration. For instance, students of the modern subject of the relation of radiation to temperature would perhaps be puzzled by § 82, which professes to give a new proof of the Stefan-Boltzmann law; the transformation of linear scale of the system æther *plus* matter, there employed, is a very tempting one, but, unfortunately, the free periods do not seem to correspond. It may be put forward as a reasonable generalisation, subject to only a few striking exceptions, that a book which can be acclaimed as free of discrepancies or obscurities is also to a large extent free of new contributions to knowledge. In the present case, the obvious advances are so important that close attention to the work throughout its whole range cannot safely be neglected.

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A STUDY IN ALPINE GEOLOGY.

Das Sonnwendgebirge im Unterinnthal. Ein Typus Alpinen Gebirgsbaues. By Dr Franz Wähner. First part. Pp. xii + 350; with 96 illustrations in the text, 19 plates and map. (Leipzig and Vienna: F. Deuticke, 1903.) Price 35 marks.

OF all the labour that has been expended on the fascinating problems of Alpine geology, none, perhaps, has been more fortunate in the manner of its presentation than the work under consideration. A lucid style, fine large type and a wealth of illustration contribute to the enjoyment of an interesting thesis. The weight and bulk of the volume, however, constitute a drawback.

The limited area dealt with by the author comprises the Haiderjoch, Rosan and the Sonnwendjoch; and the formations range from the Triassic Werfen beds to the Upper Jurassic Aptychenkalk; but it is with the rocks about the middle of this series that he is mainly concerned. These are classified in the following, descending, order:—Hornsteinkalk (upper Jura), Hornstein-Breccie, Radiolariengesteine, Rother Liaskalk [Weisser Riffkalk, Ober-rhätischer Mergelkalk, Weisser Riffkalk (lower part)], Kössen beds.

It will be recognised at once that this is an abbreviation of Pilcher's sequence. The main mass of the Weisser Riffkalk, which has all the characters of a true coral reef, has presented a difficulty to the author from the fact that he has found, in the lower parts, undoubted Rhætic fossils, and in other parts, which he considers are higher portions of the same group, Lias fossils have been discovered.

"We are so accustomed to regard the term 'Oberer Dachsteinkalk' as applied to a Rhætic rock that it does not seem wise to use it for a group which is in part Rhætic, in part Liassic."

He therefore proposes "Weisser Riffkalk" as a local term, suggestive of the salient character of the group.

Before presenting the results of his own researches, Dr. Wähner devotes the first 78 pages to the discussion of the geological literature of the Sonnwend district. Commencing with Uttinger in 1819, he passes in review practically all that has been written on the subject up to 1900 (in the preface he comments on Ampferrer's paper of 1902). On each paper he makes a few brief explanatory or critical remarks. To Dr. Diener, however, he allots some fifteen pages, occupied almost wholly in destructive criticism—"a heap of errors," he says in one place; and he is so irritated by what he regards as Diener's incorrect observations and loose writing that he waxes ironical: "I regret I cannot give any figure of this interesting spot," says Diener, which causes the author to remark,

"The reader endeavours to keep calm; perhaps D. had no time to make a sketch—but, on second thoughts, a better view is, that what Diener desires (*will*) to see, nobody *can* draw" (p. 40).

With much of the painstaking work of Pilcher, the author is in agreement, but he considers the